

# Abbey Primary School

## Whole School Written Calculation Policy

### Policy Aims:



- To ensure consistency in the teaching of written calculations across the school.
- To progressively secure pupils' understanding and confidence in using formal written methods in mathematics.
- To inform teaching staff and parents of the written mathematical methods taught at each stage of a pupils' education.

The **three main aims** of the 2014 National Curriculum for mathematics are **Fluency, Reasoning and Problem Solving**.

This policy ensures consistency and progression from the Foundation Stage to Year 6 in the written methods taught and applied when solving calculations.

**Varied Fluency:** Being able to recall facts and solve calculations with confidence and accuracy when presented in a variety of forms.

**Reasoning:** The process of using systematic steps to arrive at a conclusion about a problem and explain this as a process. Pupils are taught and encouraged to explain the steps, recognise the method through which a solution was reached and explain any mistakes or misconceptions that occur.

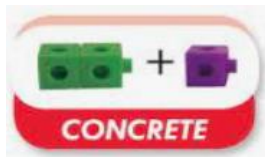
**Problem Solving:** Being able to solve mathematical problems, often involving multiple steps and operations. Problem solving requires pupils to recall key facts, have the ability to reason and be able to work systematically.

Pupils will be required to apply these written methods both when undertaking their weekly arithmetic tests and within the context of worded multi-step problems that progress in difficulty as they move up through school.



# Concrete, Pictorial, Abstract (CPA)

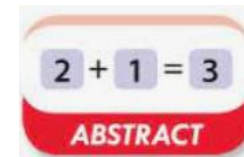
## Concrete



## Pictorial



## Abstract



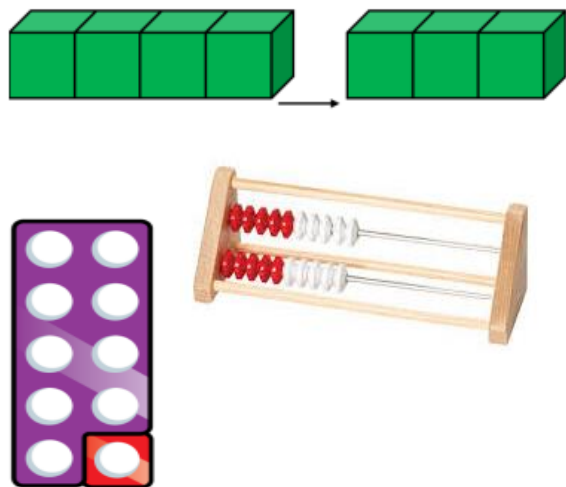
**Concrete** is the 'doing' stage, using concrete objects or equipment to solve problems – this begins with objects found in real life before specific mathematical (yet still tangible) equipment is used to represent the objects. It brings the concept to life by allowing children to handle physical objects themselves. Every new abstract concept is learned first with a 'concrete' or physical experience.

**Pictorial** is the 'seeing/visualisation' stage, using representations of the objects involved in mathematical problems. Children learn to make mental connections between the physical object and abstract levels of understanding, by drawing or looking at pictures, circles, diagrams or bar models which represent the objects in the problem.

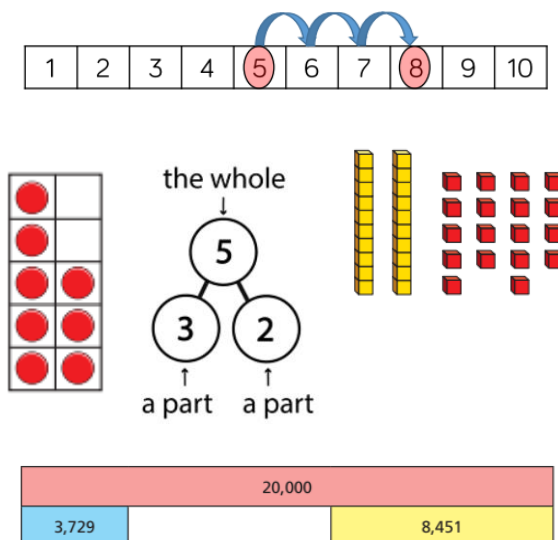
**Abstract** is the 'symbolic' stage where, once a solid understanding is demonstrated by pupils, abstract symbols are used to represent, model and solve mathematical problems.

This is where formal written recordings of calculations are used. Teachers will ensure that pupils have the opportunity to apply their knowledge in a variety of contexts and problems to deepen their understanding.

### Examples:



### Examples:



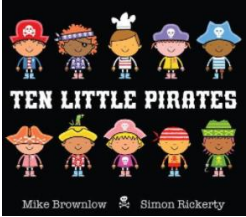
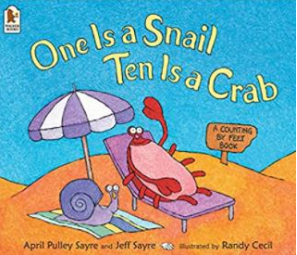
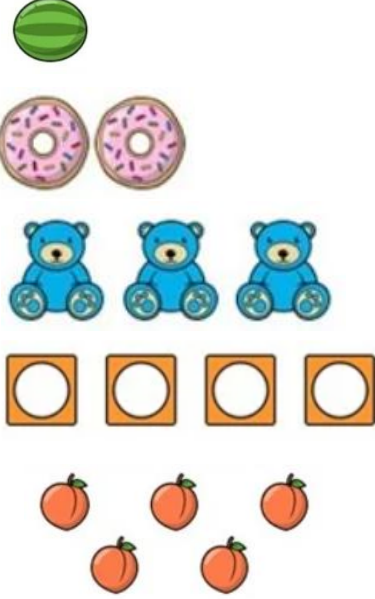
### Examples:

|   |   |   |   |
|---|---|---|---|
|   | H | T | O |
|   | 2 | 3 | 5 |
| + | 1 | 5 | 7 |
|   |   |   |   |
|   |   |   |   |

$$\begin{array}{r} 032 \\ 9 \overline{) 288} \\ \underline{18} \phantom{0} \\ 108 \\ \underline{108} \\ 0 \end{array}$$

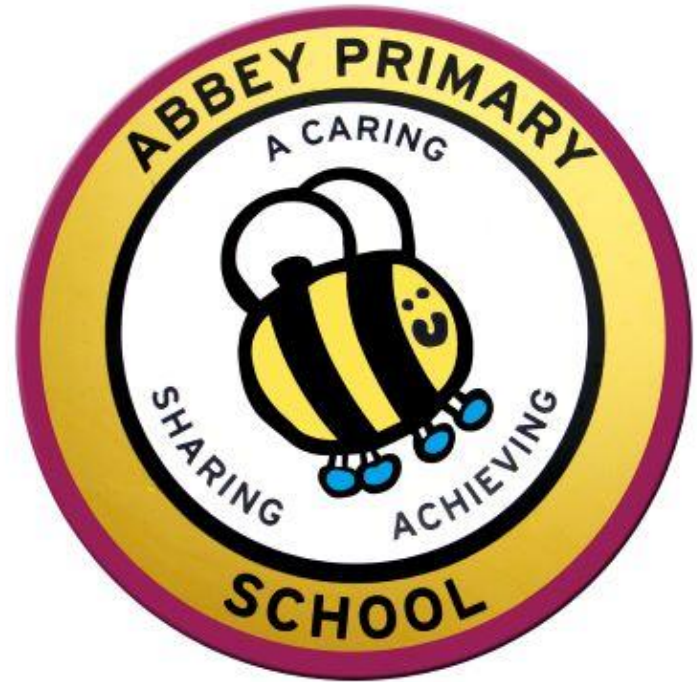
# Maths - Early Years Foundation Stage

Maths for young children should be meaningful. Where possible, concepts should be taught in the context of real life. We use a range of real-life objects alongside pictorial representations to support early counting, addition, and subtraction.

| Counting                                                                                                                                                                                                                                                                                                                                                                                     | Recognising and Matching                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | Ordering                                                                                                                                                                                       | Key Vocabulary                                                                                                                                                                                                                                                                                                                                                                                                                                                         |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>Songs, stories and games are a useful way to introduce counting.</p> <p>Example:</p> <p>Five little ducks went swimming one day:</p>    | <p>Show the numeral on your fingers:</p>  <p>Find the representation on a dice.</p>  <p>Spot the numeral in the environment, make the numeral personal and significant eg (<b>one</b> nose, <b>two</b> eyes, <b>three</b> years old etc.).</p>  <p>Make the numeral from concrete objects.</p>  <p>Recognise the numeral and the Numicon piece.</p>  <p>3<br/>three</p> | <p>Ordering different amounts of objects from smallest to largest.</p> <p>Ordering amounts of objects:</p>  | <p>Games and songs are a useful way to begin using vocabulary involved in addition and subtraction.</p> <ul style="list-style-type: none"> <li>Add</li> <li>More</li> <li>And</li> <li>Make</li> <li>Sum</li> <li>Total</li> <li>Altogether</li> <li>Score</li> <li>One more</li> <li>Smallest</li> <li>Largest</li> <li>Biggest</li> <li>Take (away)</li> <li>Leave</li> <li>Left over</li> <li>Gone</li> <li>Fewer</li> <li>Subtract</li> <li>Subtraction</li> </ul> |

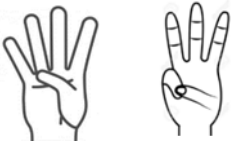
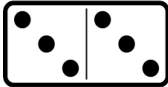
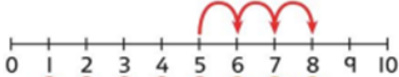
# Whole School Approach to Written Calculations

## Addition



## Early Years Foundation Stage - Addition

Maths for young children should be meaningful. Wherever possible, concepts should be taught in the context of real-life.

| Concrete                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               | Pictorial                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               | Abstract                                                                                                                                                                                                                                                                                                                                                                                          | Key Vocabulary                                                                                                                                                                                                                                                                                                   |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>Resources can be used to complete addition number sentences. This supports the understanding of numbers and what they represent:</p> <p>Numicon<br/> <math>2 + 3 =</math></p>  <p>Cubes<br/> <math>3 + 1 =</math></p>  <p>Fingers<br/> <math>4 + 3 =</math></p>  | <p>Pictorial representations are used to complete addition number sentences. This supports the understanding of numbers and what they represent.</p> <p>Dominos<br/> <math>3 + 3 =</math></p>  <p>Printed Pictures<br/> <math>2 + 2 =</math></p>  <p>Drawing Spots<br/> <math>3 + 2 =</math></p>  | <p>Resources such as number lines and number tracks can be used to count on when completing an addition number sentence.</p> <p>Number lines<br/> <math>5 + 3 =</math></p>  <p>Number Tracks<br/> <math>2 + 4 =</math></p>  | <p>Games and songs are a useful way to begin using vocabulary involved in addition.</p> <ul style="list-style-type: none"> <li>• Add</li> <li>• More</li> <li>• And</li> <li>• Make</li> <li>• Sum</li> <li>• Total</li> <li>• Altogether</li> <li>• Score</li> <li>• Double</li> <li>• One more than</li> </ul> |

## Year 1

### Combining two sets of objects (practical context)

Combining two sets of objects (aggregation) which will progress to adding more objects to a set (augmentation).

$$5+4=9$$



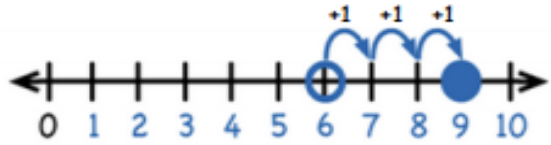
As children progress from concrete to pictorial, they continue to develop the skill of augmentation. They will begin to use the number line, number track and drawing dots (to support larger numbers up to 2 digits, but below 20).

Starting with the largest number and counting on from that, to work out the answer.

$$4+3=7$$



$$6+3=9$$



$$16+4=20$$

$$16 + \text{4 red dots} = 20$$

Ensure emphasis on the commutative nature of addition.

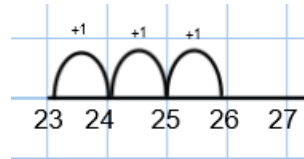
## Year 2

### Counting on a number line

As children enter Year 2, they will be fluent at using the number line. This method will be used to add a 2-digit number to a 1-digit number, until the expanded written method is taught later.

Maintain emphasis on the commutative nature of addition.

$$23+3=26$$



### Expanded written method – without bridging 10

Children apply their place value knowledge to (partitioning tens and ones) the expanded written method. Adding from the ones column first, moving to the tens and then recombining to make the total. They will use this without crossing the tens boundary until confident.

**Without crossing the tens boundary:  $33+24=58$**

$$\begin{array}{r} 30 + 3 \\ 20 + 4 \\ \hline 50 + 7 = 58 \end{array}$$

**Crossing the tens boundary:  $39+24=63$**

$$\begin{array}{r} 30 + 9 \\ 20 + 4 \\ \hline 50 + 13 = 63 \end{array}$$

**In readiness for Year 3, children are taught the expanded written method (see year 3 examples)**

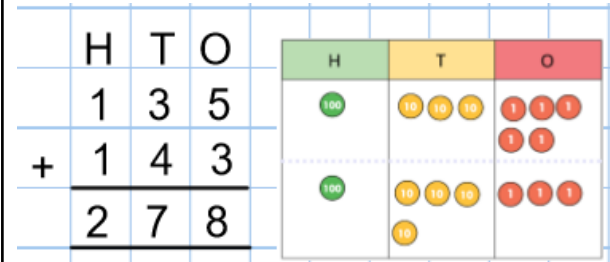
## Year 3

### Expanded written method

Upon entering year 3, children continue to use this method until they are taught the column method in the addition unit.

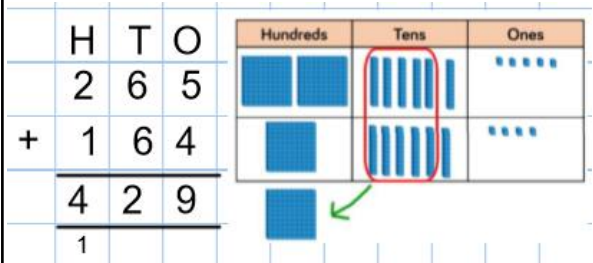
### Column method – without crossing the tens boundary

Children are introduced to the column method, modelled with place value counters or dienes. To support their place value knowledge, children to write H T O above the hundreds, tens and ones.



### Column Method – crossing the tens boundary

Children will consolidate this method throughout year 3 to ensure they are fluent and secure as they reach year 4.



## Year 4

### Compact Column Method (progressing to 4-digits)

#### Compact written method

Building precisely on the method used in Year 3, pupils continue to develop fluency using numbers with up to four digits.

|   | Th | H | T | O |
|---|----|---|---|---|
|   | 4  | 5 | 1 | 7 |
| + | 2  | 6 | 3 | 4 |
|   | 7  | 1 | 5 | 1 |
|   | 1  |   | 1 |   |

#### Using compact written method with decimals

Pupils' understanding of the number system and decimal place value is extended at this stage to tenths and then hundredths. Place value counters can be used alongside the column method to develop understanding of addition with decimal numbers if necessary.

|   | H | T | O | t |
|---|---|---|---|---|
|   |   | 7 | 2 | 8 |
| + |   | 5 | 4 | 6 |
|   | 1 | 2 | 7 | 4 |
|   |   | 1 |   |   |

## Year 5

### Compact Column Method (progressing to more than 4-digits)

#### Compact written method

Children consolidate their use of the formal method of addition, progressing to more than to 4-digits (using visual aids such as)

|   | Hth | TTh | Th | H | T | O |
|---|-----|-----|----|---|---|---|
|   | 8   | 3   | 8  | 3 | 2 | 6 |
| + | 1   | 2   | 3  | 8 | 3 | 0 |
|   | 9   | 6   | 2  | 1 | 5 | 6 |
|   |     | 1   | 1  |   |   |   |

#### Using compact written method with decimals

Pupils' apply the column method to adding decimals, including a mix of whole numbers and decimals, decimals with different numbers of decimal places, and complements of 1.  
**Children are encouraged to place a 0 as a place value holder.**

e.g.  $8.70 + 4.32 = 13.02$

|   | T | O | t | h |
|---|---|---|---|---|
|   |   | 8 | 7 | 0 |
| + |   | 4 | 3 | 2 |
|   | 1 | 3 | 0 | 2 |
|   |   | 1 |   |   |

## Year 6

### Compact Column Method (progressing to larger numbers)

As year 5 but progressing to larger numbers, aiming for both conceptual understanding and procedural fluency with columnar method to be secured.

|   | M | Hth | TTh | Th | H | T | O |
|---|---|-----|-----|----|---|---|---|
|   | 1 | 2   | 3   | 8  | 6 | 4 | 3 |
| + | 5 | 5   | 7   | 5  | 6 | 2 | 6 |
|   | 6 | 8   | 1   | 4  | 2 | 6 | 9 |
|   |   | 1   | 1   | 1  |   |   |   |

#### Using compact written method with decimals

Continue calculating with decimals, including those with different numbers of decimal places.

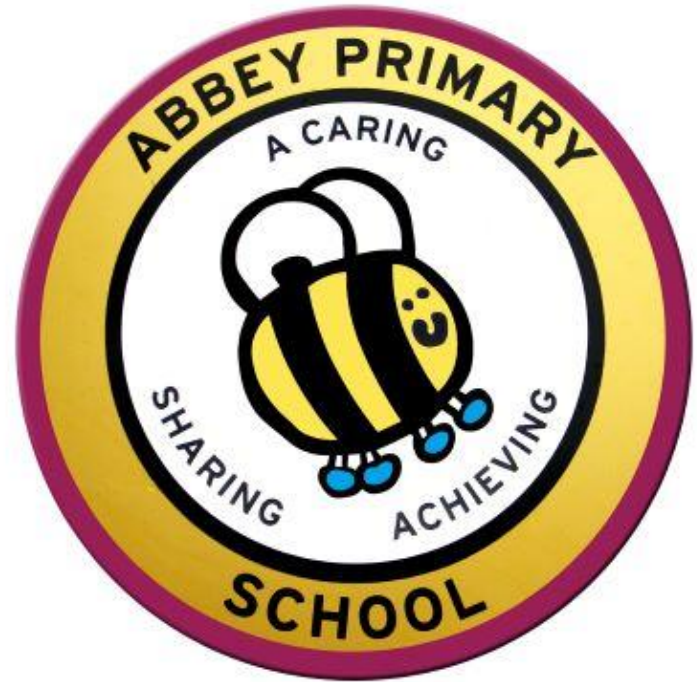
**Children are encouraged to place a 0 as a place value holder.**

e.g.  $8.870 + 6.123 = 14.993$

|   | T | O | t | h | th |
|---|---|---|---|---|----|
|   |   | 8 | 8 | 7 | 0  |
| + |   | 6 | 1 | 2 | 3  |
|   | 1 | 4 | 9 | 9 | 3  |

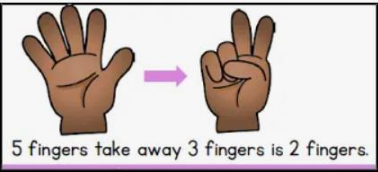
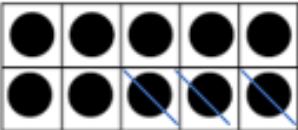
# Whole School Approach to Written Calculations

## Subtraction



## Early Years Foundation Stage - Subtraction

Maths for young children should be meaningful. Wherever possible, concepts should be taught in the context of real-life.

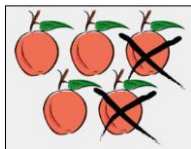
| Concrete                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | Pictorial                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               | Abstract                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | Key Vocabulary                                                                                                                                                                    |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>Resources are used to relate subtraction to 'taking away' and counting how many objects are left.</p> <p><b>Example: <math>5 - 1 = 4</math></b></p>  <p>Children can solve simple subtraction number sentences using fingers:</p> <p><b>Example: <math>5 - 3 = 2</math></b></p>  <p>5 fingers take away 3 fingers is 2 fingers.</p> | <p>Pictures can be used and crossed out to find how many are left.</p> <p><b>Example: <math>6 - 1 =</math></b></p>  <p>Children can draw pictures to help them work out a subtraction number sentence.</p> <p><b>Example: <math>9 - 3 =</math></b></p>  <p>Ten frames can also be used to help with subtraction number sentences by crossing out and counting how many are left.</p> <p><b>Example: <math>10 - 3 =</math></b></p>  | <p>Children are given subtraction number sentences to work out.</p> <p><b>Example:</b></p> <p><math>10 - 3 =</math></p> <p><math>10 - 5 =</math></p> <p>Children are given subtraction problems set in real life contexts and are encouraged to visualise the problem.</p> <p><b>Example:</b></p> <p>Mum has <b>6</b> sweets from the shop. She gave my sister <b>2</b>. How many does mum have left?</p>  | <ul style="list-style-type: none"> <li>• Take (away)</li> <li>• Leave</li> <li>• Left over</li> <li>• Gone</li> <li>• Fewer</li> <li>• Subtract</li> <li>• Subtraction</li> </ul> |

## Year 1

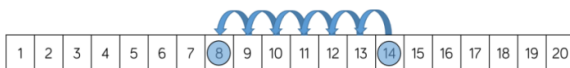
### Subtraction using objects (practical/pictorial context)

Pupils should be taught to solve one-step problems that involve subtraction using concrete objects and pictorial representations. Numbers 1-20 are used when subtracting – working up to subtracting 1-digit from 2-digit numbers.

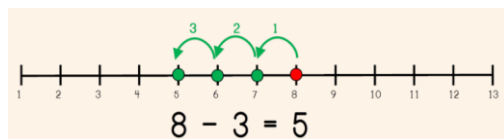
$$5 - 2 = 3$$



### Counting backwards using a numbertrack



### Counting backwards using a numberline

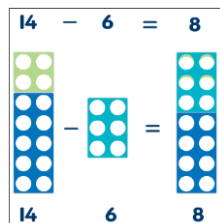


### Finding the difference (practical/pictorial context)

Understanding subtraction as finding the difference between the whole number and the part :

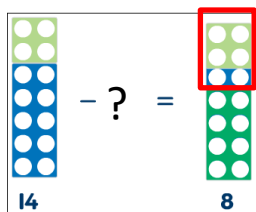
What is the difference between 14 and 8?

$$14 - 6 = 8$$



Or finding a missing Number...

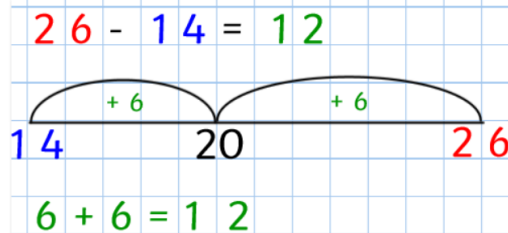
$$14 - \square = 8$$



## Year 2

### Counting backwards using a numberline (2-digit subtract 2-digit)

Building on the pupils knowledge of using a number line for subtraction, they continue to develop this method when subtracting 2-digit numbers from 2-digit numbers – as below:



Missing number problems are solved using the same method but encouraging pupils' knowledge of the inverse operation:

$$23 - \square = 18$$

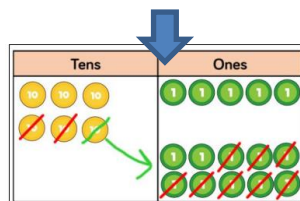
$$23 - 18 = 5$$

In readiness for Year 3, children are taught the expanded written method:

$$\begin{array}{r} 75 - 44 = 31 \\ \underline{70 - 40} \\ 30 + 1 = 31 \end{array}$$

$$\begin{array}{r} 65 - 28 = 37 \\ \underline{60 - 20} \\ 40 - 8 = 32 \\ \underline{30 + 5} \\ 37 \end{array}$$

Concrete resources are used to support children's understanding of place value.



## Year 3

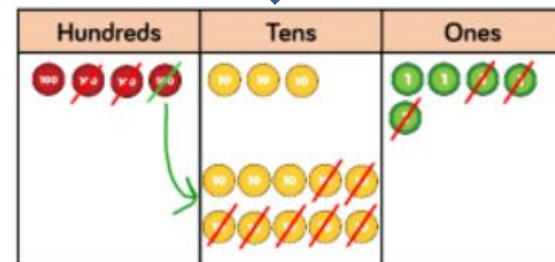
### Expanded column subtraction (progressing to 3-digits subtract 3-digit)

In year 3, pupils continue to practise the expanded column method using larger numbers up to 3-digits by 3-digits.

When the subtraction unit is taught, pupils are introduced to the compact method of subtraction, still using place value counters as a visual aid where necessary – particularly where exchanging occurs.

| H     | T | O |
|-------|---|---|
| 5     | 6 | 7 |
| 1     | 3 | 6 |
| <hr/> |   |   |
| 4     | 3 | 1 |

| H              | T  | O |
|----------------|----|---|
| <sup>3</sup> 4 | 13 | 5 |
| 2              | 7  | 3 |
| <hr/>          |    |   |
| 1              | 6  | 2 |



## Year 4

### Written methods (progressing to 4-digits)

Year 4 continue to develop fluency in using the compact column method for subtraction. Exchanges are modelled using the correct place value language. (Where needed, place value counters/dienes can be used to support understanding).

$$6,232 - 4,814 = 1,418$$

|   | T                         | H              | T                         | O              |
|---|---------------------------|----------------|---------------------------|----------------|
|   | <sup>5</sup> <del>6</del> | <sup>1</sup> 2 | <sup>2</sup> <del>3</del> | <sup>1</sup> 2 |
| - | 4                         | 8              | 1                         | 4              |
|   | 1                         | 4              | 1                         | 8              |

Progress to calculating with decimals (to include tenths and hundredths) in contexts such as money and measures.

$$5.36 - 3.80 = 1.56$$

|   | O.                        | t              | h |
|---|---------------------------|----------------|---|
|   | <sup>4</sup> <del>5</del> | <sup>1</sup> 3 | 6 |
| - | 3                         | 8              | 0 |
|   | 1                         | 5              | 6 |

Pupils are encouraged to place a 0 as a place value holder (as above).

## Year 5

### Written methods (progressing to more than 4-digits)

Children are fluent in using the compact column method of subtraction and will now begin to undertake calculations with larger numbers (more than 4 digits). (Where needed, place value counters/dienes can be used to support understanding).

Progressing to more than 4 digits:

$$38,326 - 23,830 = 14,496$$

|   | TTh | Th                        | H                          | T              | O |
|---|-----|---------------------------|----------------------------|----------------|---|
|   | 3   | <sup>7</sup> <del>8</del> | <sup>12</sup> <del>3</del> | <sup>1</sup> 2 | 6 |
| - | 2   | 3                         | 8                          | 3              | 0 |
|   | 1   | 4                         | 4                          | 9              | 6 |

Progress to calculating with decimals with different numbers of decimal places (up to 3 d.p)

$$1.001 - 0.520 = 0.481$$

|   | O.                        | t                         | h              | th |
|---|---------------------------|---------------------------|----------------|----|
|   | <sup>0</sup> <del>1</del> | <sup>9</sup> <del>0</del> | <sup>1</sup> 0 | 1  |
| - | 0                         | 5                         | 2              | 0  |
|   | 0                         | 4                         | 8              | 1  |

Pupils are encouraged to place a 0 as a place value holder (as above).

## Year 6

### Written methods

In year 6, pupils use the compact column method of subtraction to calculate even larger numbers, aiming for conceptual understanding, procedural fluency and accuracy.

$$389,949 - 150,699 = 239,250$$

|   | Hth | TTh | Th | H                         | T               | O |
|---|-----|-----|----|---------------------------|-----------------|---|
|   | 3   | 8   | 9  | <sup>8</sup> <del>9</del> | <sup>14</sup> 4 | 9 |
| - | 1   | 5   | 0  | 6                         | 9               | 9 |
|   | 2   | 3   | 9  | 2                         | 5               | 0 |

Pupils continue calculating with decimal numbers linking to various concepts and measures.

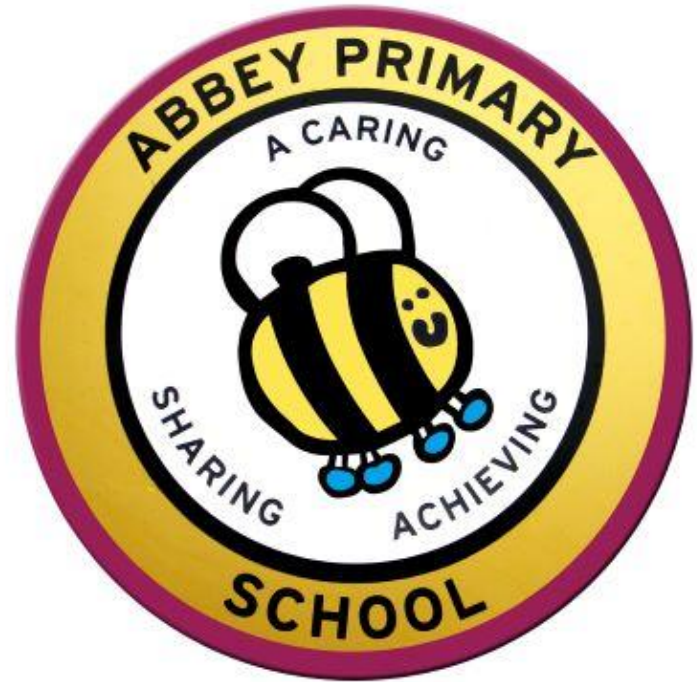
$$12.7 - 1.953 = 10.747$$

|  | T | O.                        | t                          | h                         | th             |
|--|---|---------------------------|----------------------------|---------------------------|----------------|
|  | 1 | <sup>1</sup> <del>2</del> | <sup>16</sup> <del>7</del> | <sup>9</sup> <del>0</del> | <sup>1</sup> 0 |
|  | 0 | 1                         | 9                          | 5                         | 3              |
|  | 1 | 0                         | 7                          | 4                         | 7              |

Pupils are encouraged to place a 0 as a place value holder (as above).

# Whole School Approach to Written Calculations

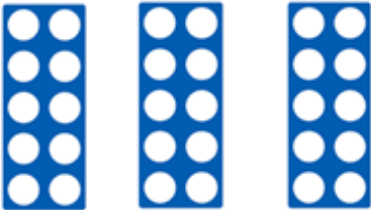
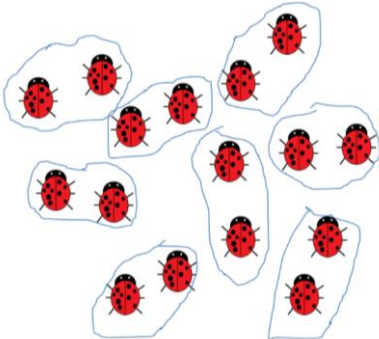
## **Multiplication**



## Early Years Foundation Stage - Multiplication

Maths for young children should be meaningful. Wherever possible, concepts should be taught in the context of real-life.

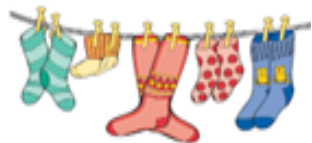
The link between addition and multiplication is introduced through repeated addition.

| Concrete                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | Pictorial                                                                                                                                                                                                                                                                                                                                                                                                                                                 | Key Vocabulary                                                                                                 |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------|
| <p>Children are taught to understand multiplication as repeated addition.</p> <p>What are 2 groups of 2?</p>  <p><math>2 + 2 = 4</math></p> <p>Count in 2s, 5s and 10s both aloud (by rote) and with Numicon.</p>  <p>How many legs do 3 ducks have?</p> <p><math>2 + 2 + 2 = 6</math><br/>There are 6 legs.</p> | <p>Real life contexts and use of pictures to count in repeated groups of the same size.</p> <p><b>Example:</b></p>  <p>What are 3 groups of 10?<br/><math>10 + 10 + 10 = 30</math><br/>3 groups of 10 are 30</p> <p>Group 16 ladybirds into 2s. How many groups will there be?</p>  | <ul style="list-style-type: none"><li>• Lots of</li><li>• Groups of</li><li>• Times</li><li>• Double</li></ul> |

## Year 1

### Repeated Addition (practical context)

Children are taught to understand multiplication as repeated addition.



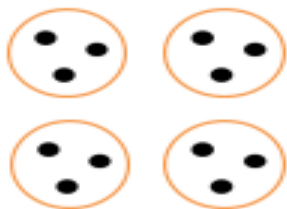
$$2+2+2+2+2=10$$

(5 groups of 2)



$$5+5+5+5+5+5=30$$

(6 groups of 5)

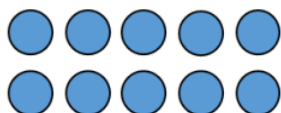


$$3+3+3+3=12$$

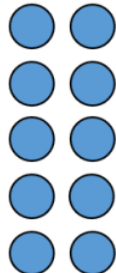
(4 groups of 3)

### Arrays

Using arrays helps pupils to understand that multiplication can be done in any order (commutative). The children are encouraged to count in groups.



$$2+2+2+2+2=10$$

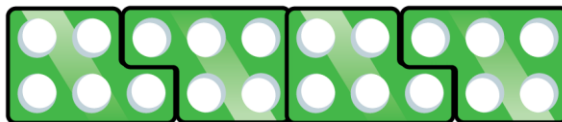
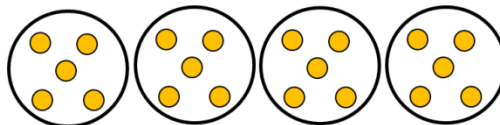


$$5+5=10$$

## Year 2

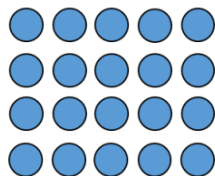
In Year 2, children are introduced to the multiplication symbol alongside pictorial representations:

e.g.  $5+5+5+5=20$  and  $5 \times 4=20$



### Arrays (continued)

Children continue to use arrays to reinforce that multiplication can be done in any order (commutative). The children are encouraged to count in groups as below so that they can see the commutative nature of multiplication.



$$5+5+5+5=20$$

$$4 \times 5=20$$

$$5 \times 4=20$$

In the summer term of year 2, children are introduced to the more formal grid method of multiplication in readiness for entry to Year 3:

|   |    |    |
|---|----|----|
| X | 10 | 2  |
| 5 | 50 | 10 |



|   |   |   |
|---|---|---|
| H | T | O |
|   | 5 | 0 |
| + | 1 | 0 |
|   | 6 | 0 |

## Year 3

Children will continue to develop their understanding and use of the grid method to complete larger multiplication calculations (2 digit by 1 digit).

|   |    |    |
|---|----|----|
| X | 20 | 4  |
| 3 | 60 | 12 |



|   |   |   |
|---|---|---|
| H | T | O |
|   | 6 | 0 |
| + | 1 | 2 |
|   | 7 | 2 |

In the Spring Term, when multiplication of 2-digit by one-digit numbers is taught, pupils will be introduced and to the expanded column method of multiplication, as below.

|   |   |   |   |          |  |
|---|---|---|---|----------|--|
|   | H | T | O |          |  |
|   |   | 3 | 4 |          |  |
| x |   |   | 5 |          |  |
|   |   | 2 | 0 | (5 x 4)  |  |
| + | 1 | 5 | 0 | (5 x 30) |  |
|   | 1 | 7 | 0 |          |  |

## Year 4

Pupils entering Year 4 will be confident at using the expanded grid method of multiplication to carry out 2-digit by 1-digit calculations.

|   | H | T | O |          |  |
|---|---|---|---|----------|--|
|   |   | 3 | 4 |          |  |
| x |   |   | 5 |          |  |
|   |   | 2 | 0 | (5 × 4)  |  |
| + | 1 | 5 | 0 | (5 × 30) |  |
|   | 1 | 7 | 0 |          |  |

The same (expanded) method will be applied and mastered for the duration of the year using larger numbers (up to 3-digit by 1-digit). Pupils should enter Year 5 using this method confidently to support their understanding of place value.

|   | H | T | O |           |
|---|---|---|---|-----------|
|   | 4 | 2 | 8 |           |
| x |   |   | 6 |           |
|   |   | 4 | 8 | (6 × 8)   |
|   | 1 | 2 | 0 | (6 × 20)  |
| 2 | 4 | 0 | 0 | (6 × 400) |
| 2 | 5 | 6 | 8 |           |

## Year 5

As pupils enter Year 5, they will be fluent in using the expanded column method for multiplication and should therefore be introduced to the compact method of multiplication, as below.

|   | H | T | O |           |
|---|---|---|---|-----------|
|   | 4 | 2 | 8 |           |
| x |   |   | 6 |           |
|   |   | 4 | 8 | (6 × 8)   |
|   | 1 | 2 | 0 | (6 × 20)  |
| 2 | 4 | 0 | 0 | (6 × 400) |
| 2 | 5 | 6 | 8 |           |

|   | H | T | O |
|---|---|---|---|
|   | 4 | 2 | 8 |
| x |   |   | 6 |
| 2 | 5 | 6 | 8 |
|   | 1 | 4 |   |

Pupils will become more fluent and confident in using the compact method of column multiplication for calculations up to 4-digit by 2-digit.

|   |   | Th | H | T | O |
|---|---|----|---|---|---|
|   |   | 7  | 5 | 2 | 3 |
|   | x |    |   | 2 | 4 |
|   | 3 | 0  | 0 | 9 | 2 |
| 1 | 5 | 0  | 4 | 6 | 0 |
| 1 | 8 | 0  | 5 | 5 | 2 |
|   |   |    |   |   |   |

## Year 6

Pupils enter Year 6 securely using the compact method of column multiplication.

|   |   | T <sub>h</sub> | H | T | O |
|---|---|----------------|---|---|---|
|   |   | 7              | 5 | 2 | 3 |
|   | x |                |   | 2 | 4 |
|   | 3 | 0              | 0 | 9 | 2 |
| 1 | 5 | 0              | 4 | 6 | 0 |
| 1 | 8 | 0              | 5 | 5 | 2 |
|   |   |                |   |   |   |

### Multiplying a Decimal Number by an Integer

Introduce multiplying decimals with up to 2 decimal places by a whole number (integer).

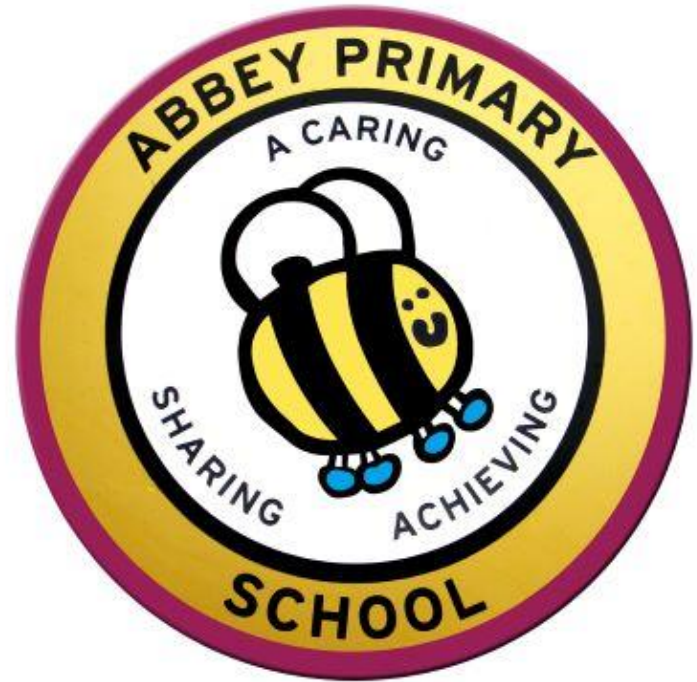
|   | O | Tt | Ht |
|---|---|----|----|
|   | 3 | 1  | 9  |
| x | 8 |    |    |
| 2 | 5 | 5  | 2  |
|   |   |    |    |

Remind children that the whole number (integer) belongs in the ones column because it is a whole number. Line up the decimal points in the question and the answer.

Use this for money and other measures.

# Whole School Approach to Written Calculations

## Division



## Early Years Foundation Stage - Division

Maths for young children should be meaningful. Wherever possible, concepts should be taught in the context of real-life.

Children see, hear and experience representations of division as both grouping and sharing.

| Concrete                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | Key Vocabulary                                                                                                                                         |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>By sharing quantities equally/fairly in practical contexts, pupils begin to understand division.</p> <p><b>In the home corner:</b> There are 10 chopsticks – can you share them between the people around the table equally?</p>  <p><b>Example:</b></p> <p>6 cubes shared into 2 equal groups = 3 cubes</p>  | <ul style="list-style-type: none"><li>• Halve</li><li>• Share</li><li>• Share Equally</li><li>• One Each, Two Each</li><li>• Equal Groups Of</li></ul> |

## Year 1

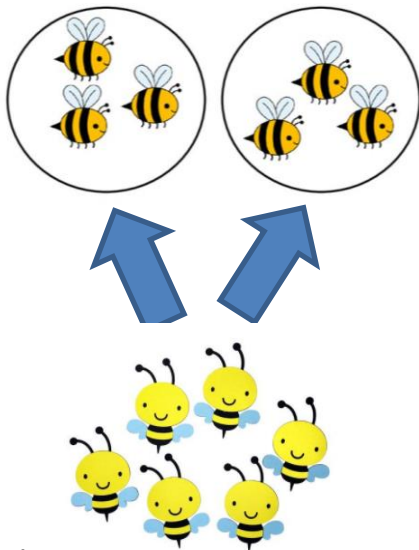
### Sharing and Grouping (practically with objects)

Through grouping and sharing quantities in practical contexts, pupils begin to understand division.

#### Sharing

Children learn to share objects equally in a variety of contexts as below:

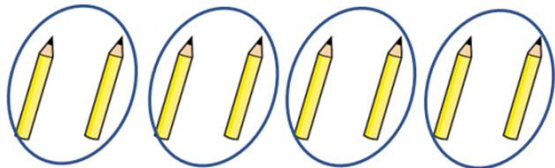
There are 6 bees and they are shared between 2 bee keepers. How many bees does each beekeeper get?



#### Grouping

Children apply their counting skills to develop their understanding of grouping. Children move objects into groups of a suggested amount before checking that they are all equal.

Make equal groups of 2



## Year 2

In Year 2, children are introduced to the division symbol alongside pictorial representations, as below:

#### Sharing using the Singapore bar:

How many groups of 5 are in 15?



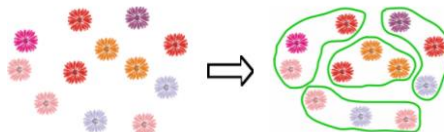
$$15 \div 3 = 5$$

Children use a Singapore Bar to divide a number into equal groups. Dots represent the whole number to be shared equally between each part of the bar in turn.

#### Grouping a Whole Number

Children learn to separate an amount into groups of a suggested number, as below:

Steven has 20 flowers and wants to put them into groups of 3.



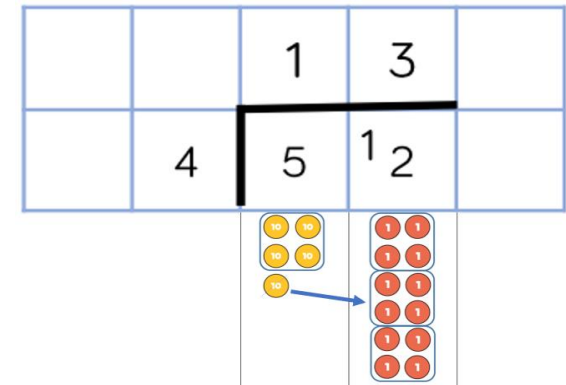
How many groups of 3 flowers will he have?

To be introduced in the summer term of Year 2 in preparation for Year 3:

## Year 3

### Short Division Method

Formal short division should now be introduced. When being introduced to this method, children build on their understanding of grouping using place value counters or Dienes alongside. Starting with the largest place value, they group by the divisor, as below:



How many groups of 4 tens are there in 5 tens? **1 group**

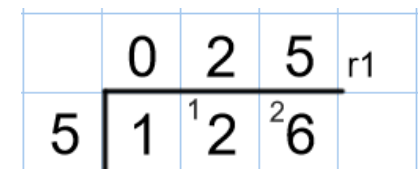
How many tens are left over? **1 ten**

This is then exchanged into the ones column (exchanged for ten ones).

Now, how many groups of 4 ones are there?

**3 groups**

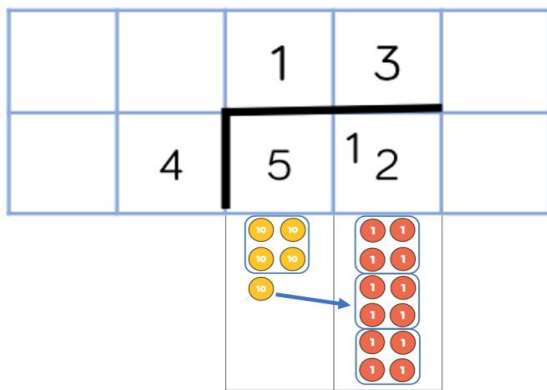
As pupils' fluency and understanding develops, they will begin to divide numbers that will result in a remainder (numbers up to 3-digits by 1-digit).



## Year 4

### Short Division Method

Formal short division will be recapped. Children build on their understanding of grouping using place value counters or Dienes alongside. Starting with the largest place value, they group by the divisor, as below:



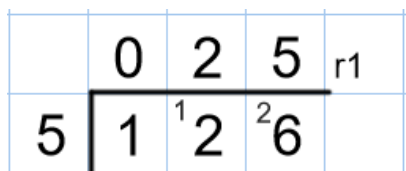
How many groups of 4 tens are there in 5 tens? **1 group**

How many tens are left over? **1 ten**

This is then exchanged into the ones column (exchanged for ten ones).

Now, how many groups of 4 ones are there? **3 groups**

As pupils' fluency and understanding develops, they will begin to divide numbers that will result in a remainder (numbers up to 3-digits by 1-digit).

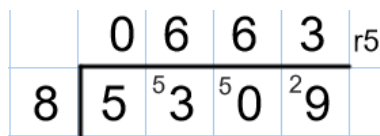


## Year 5

### Formal Written Methods

Children continue to use the formal method for short division, dividing numbers up to 4-digits by a 1-digit number and interpreting remainders appropriately.

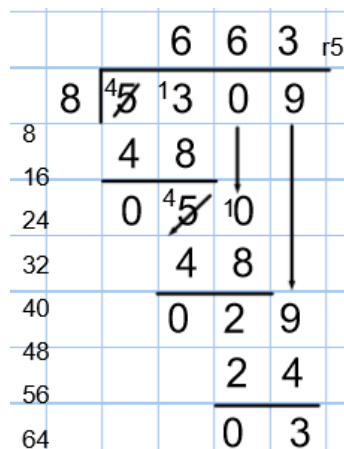
$$5309 \div 8 = 663 \text{ r } 5$$



\*For the purpose of the arithmetic test, calculations with a two-digit divisor will also be introduced in Year 5.

Children should begin to consider the meaning of the remainder in a real life problem solving context i.e. as a fraction, a decimal or as a rounded number or value depending on the context of the problem.

As pupils' fluency and understanding develops, they will begin to use the long division method (see below), in readiness for year 6. They need to be secure with this method entering their SATs year.



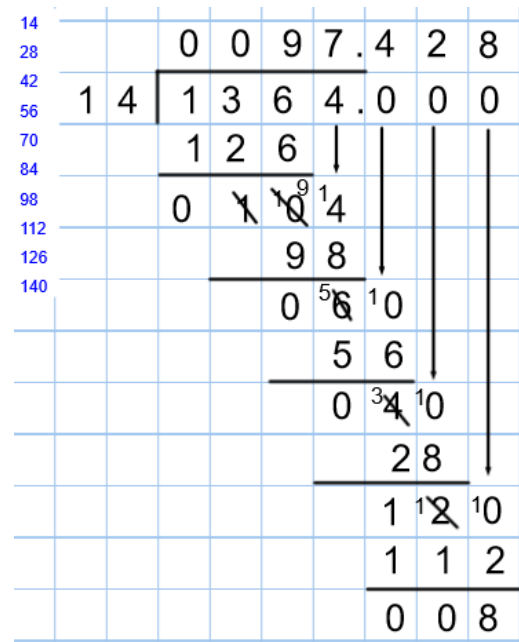
## Year 6

### Formal Written Methods – long division

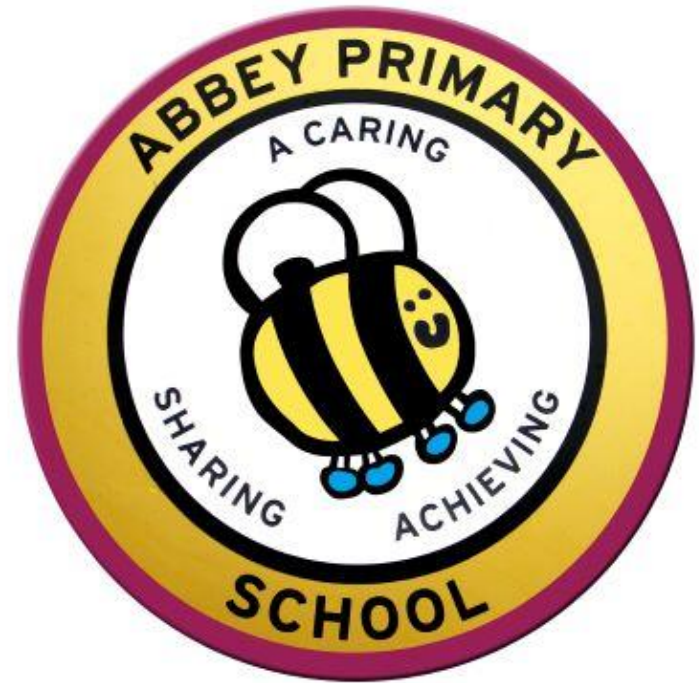
As children enter year 6, they need to be secure with applying the long division method, dividing up to 4-digits by a 2-digit number, interpreting any remainders according to the context, with up to 3 decimal places.

Children are encouraged to write the multiples of the divisor before attempting the long division.

$$\begin{aligned} 1364 \div 14 &= 97.428 \quad (3 \text{ decimal places}) \\ &= 97 \text{ r } 6 \quad (\text{remainder}) \\ &= 97 \frac{6}{14} \text{ or } \frac{3}{7} \quad (\text{as a fraction}) \end{aligned}$$



# Whole School Approach to Written Calculations



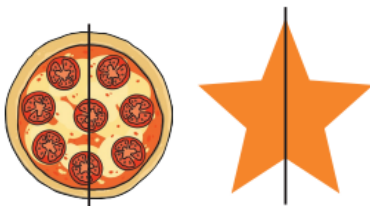
## Fractions

## Year 1

### Shape

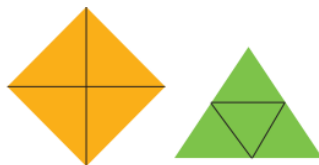
These objects and shapes are split in half. Each whole has 2 equal parts.

$$\frac{1}{2} = \text{one half}$$



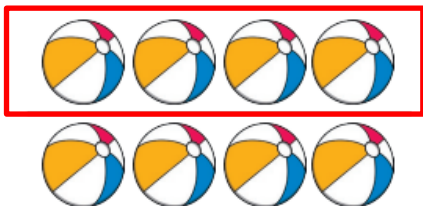
These shapes are split into quarters. Each whole has 4 equal parts.

$$\frac{1}{4} = \text{one quarter}$$

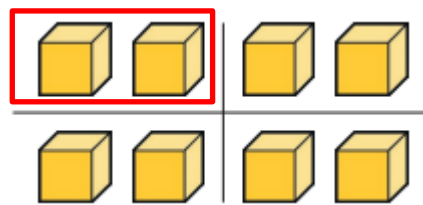


### Object

There are 8 balls. They are grouped into 2 equal groups. Half of 8 is 4.



There are 8 blocks. 8 blocks are grouped into 4 equal groups. A quarter of 8 is 2.



## Year 2

Pupils progress from recognising half and quarters, to recognising  $\frac{1}{3}$ ,  $\frac{1}{4}$ ,  $\frac{2}{4}$  of a length, shape, set of objects or quantity.

Pupils begin to recognise the equivalence of  $\frac{1}{2}$  and  $\frac{2}{4}$ .

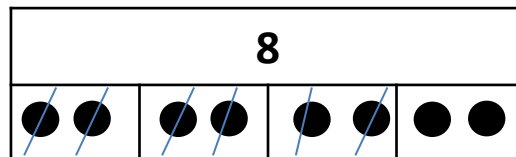
$$\frac{1}{2} = \frac{2}{4}$$



### Fraction of an amount

$$\frac{3}{4} \text{ of } 8 = 6$$

Pupils use (and draw) the Singapore bar, using the denominator to understand how many to split the bar into e.g., 4 (quarters). The whole amount is shared equally between the parts. Finally, they look at the numerator to see how many parts they need to count e.g., 3 out of 4 parts.



## Year 3

Pupils are taught to add, subtract and compare fractions with the same denominator **within one whole**.

$$\frac{2}{5} + \frac{1}{5} = \frac{3}{5}$$



$$\frac{5}{6} - \frac{2}{6} = \frac{3}{6}$$



### Fractions of an amount

Pupils are introduced to finding the fraction of an amount using the following method.

$$\frac{2}{5} \text{ of } 40 = 16$$

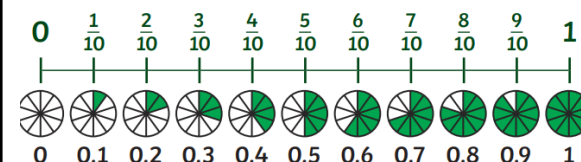
Divide the integer by the denominator:

$$40 \div 5 = 8$$

Then, multiply the answer by the numerator

$$8 \times 2 = 16$$

### Counting in tenths



## Year 4

Once pupils are fluent in addition and subtraction of fractions with the same denominator. They begin to add and subtract fractions beyond a whole e.g.

$$\frac{4}{5} + \frac{2}{5} = \frac{6}{5}$$

Addition:



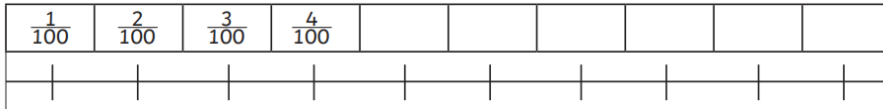
$$\frac{8}{6} - \frac{5}{6} = \frac{3}{6}$$

Subtraction:



### Hundredths

Pupils count up and down in hundredths:



They recognise that hundredths arise when dividing an object by one hundred and dividing tenths by 10, using practical resources and drawn models to support their understanding.

### Fractions of an amount

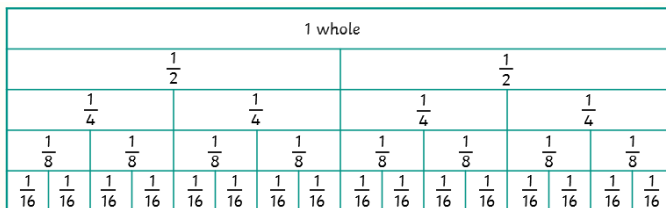
Pupils continue to develop their understanding and fluency of working out fractions of amounts (unit and non-unit fractions).

$$\frac{5}{8} \text{ of } 56 = 35$$

$$56 \div 8 = 7$$

$$7 \times 5 = 35$$

**Recognise and show, using diagrams, families of common equivalent fractions**



## Year 5

Pupils begin to recognise improper fractions and convert these into mixed numbers and vice versa.

### Multiplying fractions

Pupils are taught to multiply fractions by multiplying the numerators together and then multiplying the denominators together to calculate.

$$\frac{2}{7} \times \frac{2}{4} = \frac{4}{28}$$

### Multiplying fractions by integers

Pupils are taught to multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams.

Pupils are taught to recognise the integer as fraction by representing as parts of a whole as below:

$$\frac{3}{4} \times 3 = \frac{9}{4} = 2 \frac{1}{4}$$

Step 1:  $\frac{3}{4} \times \frac{3}{1}$

Step 2:  $\frac{3}{4} \times \frac{3}{1} = \frac{9}{4}$

### Fractions, Decimals and Percentages

Pupils are taught to recognise the relationship between fractions (100ths), decimals and percentages.

**Mixed numbers and improper fractions**

Mixed numbers and improper fractions are two different ways of writing fractions that are greater than one, or a whole.

Mixed numbers contains a whole number and a fraction part.

An improper fractions have numerators that is greater than or equal to the denominator.

**Converting Improper fractions to a mixed number.**

$$\frac{9}{4}$$

1. Divide the **numerator** by the **denominator**.

4 goes into 9 twice which 1 left over, therefore:

$$9 \text{ divided by } 4 = 2 \text{ remainder } 1$$

2. You now have 2 wholes and 1 remainder which becomes the **numerator**. The **denominator** **does not change**.

$$2\frac{1}{4}$$

3. Therefore:

$$\frac{9}{4} = 2\frac{1}{4}$$

**Converting mixed number fraction to improper fractions**

$$3\frac{1}{2}$$

1. Multiply the **denominator** by the whole number.

$$2 \times 3 = 6$$

2. Add this number to the **numerator**. This creates:

$$6 + 1 = 7 \quad \frac{7}{2}$$

3. Therefore:

$$3\frac{1}{2} = \frac{7}{2}$$

Adding fractions**Adding fractions with different denominators:**

Children write the common multiples under the denominators, until they find the lowest common denominator. Remembering to multiply the denominator and the numerator by the same amount.

$$\frac{1}{3} \begin{matrix} \times 4 \\ \times 4 \end{matrix} + \frac{1}{4} \begin{matrix} \times 3 \\ \times 3 \end{matrix} + \frac{1}{2} \begin{matrix} \times 5 \\ \times 5 \end{matrix}$$

Step 1:

$$\begin{array}{ccc} 6 & 8 & 4 \\ 9 & 8 & 6 \\ \boxed{12} & \boxed{12} & \boxed{12} \end{array}$$

Step 2:

$$\frac{4}{12} + \frac{3}{12} + \frac{6}{12} = \frac{13}{12} = 1 \frac{1}{12}$$

Subtracting

Children to use the same method with finding the lowest common denominator, so that they can subtract the numerator.

$$\frac{6}{7} - \frac{6}{14}$$

Step 1:

$$\frac{6}{7} \begin{matrix} \times 2 \\ \times 2 \end{matrix}$$

Step 2:

$$\frac{12}{14}$$

Step 3:

$$\frac{12}{14} - \frac{6}{14} = \frac{6}{14}$$

Adding fractions with mixed numbers

$$3 \frac{9}{10} + \frac{1}{2} =$$

The mixed number is converted to an improper fraction by multiplying the denominator by the integer.

Step 1:

$$\frac{39}{10} + \frac{1}{2}$$

Step 2:

$$\frac{39}{10} + \frac{5}{10} = \frac{44}{10} = 4 \frac{4}{10}$$

Dividing fractions by an integer

$$\frac{3}{4} \div 3 = \frac{3}{12}$$

Step 1:

$$\frac{3}{4} \div \frac{3}{1}$$

Step 2:

$$\frac{3}{4} \div \frac{1}{3}$$

Step 3:

$$\frac{3}{4} \times \frac{1}{3} = \frac{3}{12}$$

1. Place the integer over 1 to form a fraction.

2. Flip the fraction around so that the 1 becomes the numerator.

3. Switch the division to a multiplication.

4. Multiply the numerators together and then the denominators.